Preprint typeset in JHEP style - HYPER VERSION

The Higgs field and the ultraviolet behaviour of the vortex operator in 2+1 dimensions

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Abstract: We calculate the change in the ultraviolet behaviour of the vortex operator due to the presence of dynamical Higgs field in both 2+1 dimensional QED and the 2+1 dimensional Georgi-Glashow model. We find that in the QED case the presence of the Higgs field leads at the one loop level to power like correction to the propagator of the vortex operator. On the other hand, in the Georgi-Glashow model, the adjoint Higgs at one loop has no affect on the vortex propagator. Thus, as long as the mass of the Higgs field is much larger than the gauge coupling constant, the ultraviolet behaviour of the vortex operator in the Georgi-Glashow model is independent of the Higgs mass.

Contents

1. Introduction

During the last several years the vortex mechanism of confinement has seen a steady increase in popularity. Originally introduced by 't Hooft in 1978 [1] it has been for a long time eclipsed by the rival monopole, or dual superconductor mechanism [2]. The dual superconductor mechanism has attracted a lot of attention during the 80's and most of the 90's. Nevertheless, a steady background of work on the understanding of magnetic vortices both on the lattice [3] and in the continuum [4] has been maintained. More recently the interest in the vortex mechanism has been revived in the lattice community by the series of works by Greensite and collaborators [5]. The notion that magnetic vortices are important for confinement seems to have earned credibility even among the long time proponents of the monopole condensation mechanism [6].

Ideally, of course, rather than just performing numerical studies, one would like to have a simple theoretical description of the relevant confining dynamics directly in terms of the operators that create and annihilate magnetic vortices. While in 3+1 dimensions this goal has not been achieved, in 2+1 dimensional confining theories such a description exists [7]. The effective low energy description in terms of the vortex operators is well established in the weakly coupled confining theories in 2+1 dimensions. It provides a simple and intuitive picture of confinement and also deconfining phase transition at finite temperature [8].

It has been argued that the main properties of the vortex operators and the confinement mechanism carry over to the strongly interacting theories, like pure gluodynamics [9]. The reason is that in 2+1 dimensions the putative Higgs phase with a perturbatively massless photon is not separated by a phase transition from a confining phase. One therefore can imagine taking the pure gluodynamics limit by starting with the weakly coupled Georgi-Glashow model, where the confining physics is well understood, and then continuously changing the coefficient of the Higgs mass term so that it becomes positive and then very large. In this transition between the Higgs regime and the confinement regime one should

not encounter any non-analyticity. It is still far from straightforward to study the vortex operators in such a theory, since their infrared physics is genuinely nonperturbative. On the other hand, the ultraviolet physics in these theories is perturbative. It is thus interesting to see what can be said about the ultraviolet properties of the vortex operator, such as its scaling behaviour at short distances, and in particular what are the effects of decoupling the Higgs field on it.

This is the question we ask in this paper: *what is the influence of dynamical matter fields on the ultraviolet behaviour of the vortex operator $V(x)$?* To answer this we study two 2+1 dimensional gauge theories: noncompact quantum electrodynamics (QED) with a charged scalar field, and the $SU(2)$ gauge theory with the Higgs field in the adjoint representation. In the former theory confinement is logarithmic rather than linear. Nevertheless the vortex operator plays a crucial role in the low energy dynamics [4]. The latter theory in its weakly coupled incarnation is the simplest known model which exhibits linear confinement between fundamental charges [10]. In this weakly coupled regime, where classically the Higgs field has a nonvanishing expectation value, this model is usually known as the Georgi-Glashow model. We are interested in the opposite regime, namely when the VEV of the Higgs vanishes and their masses become large. Nevertheless, since the two regimes are analytically connected in 2+1 dimensions, we will continue to call this theory the Georgi-Glashow model.

The regime of interest to us is when the mass of the Higgs field is much larger than the gauge coupling constant ($M \gg g^2$), and the distance between the vortex and the antivortex in the correlation function is comparable to M^{-1}. We will see that the effect of the Higgs field is quite different in the two models. In QED the dynamical Higgs field affects the ultraviolet behaviour of the vortex operator at the one loop level and leads to a power like factor in the vortex correlation function:

$$\langle V(x)V^*(y)\rangle = \left(\frac{1}{|x-y|^2\Lambda^2}\right)^{\frac{1}{8}} \langle V(x)V^*(y)\rangle_0 \;, \tag{1.1}$$

where

$$\langle V(x)V^*(y)\rangle_0 = \exp\left(-\frac{\pi}{g^2}[\Lambda - \frac{1}{|x-y|}]\right) \tag{1.2}$$

is the vortex correlator in the theory without Higgs.

In the Georgi-Glashow model, on the other hand, it turns out that the adjoint Higgs has no effect on the vortex correlator in this regime. In this sense the pure gluodynamics regime in the Georgi-Glashow model is achieved very efficiently, since the moment the Higgs mass is large, the vortex correlation function has its pure gluodynamics behaviour at all distance scales below g^{-2}, including the true ultraviolet regime.

We start our discussion with the Abelian theory.

2. The noncompact $U(1)$ model

Consider the $U(1)$ gauge theory with a complex scalar matter field defined by the (Euclidean) Lagrangian

$$L = \frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + igA_\mu)\phi|^2 + M^2\phi^*\phi \,. \tag{2.1}$$

The construction of the vortex operator and its role in the low energy dynamics of this theory has been extensively reviewed recently. The vortex operator is defined as

$$V(x) = \exp\left\{\frac{2\pi i}{g}\int_{C(x)} E_i(y)\epsilon_{ij}dy_j\right\}\,, \tag{2.2}$$

where the contour $C(x)$ starts at the point x and goes to infinity. The important properties of V are:

i) $V(x)$ does not depend on the curve C;

ii) $V(x)$ is a local, gauge invariant scalar field.

These may not be immediately obvious from the definition eq.(2.2), but nevertheless they have been rigorously established (see [7] for review).

The expectation value of V in the Euclidean path integral formalism is given by the following expression

$$\langle V(x)\rangle = Z^{-1}\int[dA_\mu][d\phi]\exp - \int d^3x\left(\frac{1}{4}(F_{\mu\nu}-s_{\mu\nu})^2 + |(\partial_\mu + igA_\mu)\phi|^2 + M^2\phi^*\phi\right), \tag{2.3}$$

where Z is the normalization factor in the partition function of the theory and the c-number source function $s_{\mu\nu}$ is given by

$$s_{\mu\nu} = \frac{2\pi}{g}\epsilon_{\mu\nu\lambda}\tau_\lambda(C)\delta^2(x\in C)\,, \tag{2.4}$$

with $\tau_\lambda(C)$ the unit vector tangent to the contour C. Due to the explicit presence of $1/g$ in the source, one can calculate the path integral eq.(2.3) in the steepest descent approximation. To leading order the presence of the Higgs field is irrelevant. The solution of the classical equations of motion in the presence of the source $s_{\mu\nu}$ is the Dirac monopole configuration with the Dirac string along C. The action of this solution is

$$S_{\text{classical}} = \frac{\pi}{2g^2}\,\Lambda \tag{2.5}$$

and the corresponding vortex VEV is

$$\langle V\rangle_0 = e^{-\frac{\pi}{2g^2}\Lambda}\,, \tag{2.6}$$

where Λ is the ultraviolet cutoff. The same calculation for the vortex-antivortex correlation function gives

$$\langle V(x)V^*(y)\rangle_0 = e^{-\frac{\pi}{g^2}[\Lambda - \frac{1}{|x-y|}]}\,. \tag{2.7}$$

The dynamical Higgs field enters the calculation of the VEV at one-loop resulting in

$$\langle V\rangle = e^{-\frac{\pi}{2g^2}\Lambda}\,\mathrm{Det}^{-1}\left(\frac{-|D|^2+M^2}{-\partial^2+M^2}\right)\,, \tag{2.8}$$

where D_μ is the covariant derivative in the classical field of a Dirac monopole. The rest of this section is devoted to the calculation of the determinant in eq.(2.8).

The contribution due to the determinant is infrared finite, but ultraviolet divergent. Since the ultraviolet cutoff Λ and the Higgs mass are the only scales in the calculation, the result must be

$$\ln \mathrm{Det}\left(\frac{-|D|^2+M^2}{-\partial^2+M^2}\right)=\mathrm{tr}\ln\left(\frac{-|D|^2+M^2}{-\partial^2+M^2}\right)=\alpha\ln\left(\frac{\Lambda^2}{M^2}\right), \tag{2.9}$$

with α a pure number and we neglect corrections with positive powers in M/Λ. Our aim is to calculate this number α. For convenience we will consider the derivative of eq.(2.9) with respect to M^2:

$$\mathrm{tr}\left[\frac{1}{-|D|^2+M^2}-\frac{1}{-\partial^2+M^2}\right]=-\alpha M^{-2}\,. \tag{2.10}$$

This calculation is equivalent to solving the quantum mechanical problem of a scalar particle in the field of the Dirac monopole of unit magnetic charge. Therefore, consider a quantum mechanical Hamiltonian

$$H=-D^*D+\mathcal{V}(r) \tag{2.11}$$

and the associated time-independent Schrödinger equation

$$H\Psi=\epsilon\Psi\,. \tag{2.12}$$

For any rotationally invariant potential $\mathcal{V}(r)$ the angular part of the problem is solved by separation of variables:

$$\Psi=f_l(r)Y^q_{l,m}(\theta,\phi)\,, \tag{2.13}$$

where Y^q are the so-called monopole harmonics [11], [12] corresponding to a magnetic monopole of magnetic charge $4\pi q/g$ and are analogous to the ordinary spherical functions. In our case $q=1/2$ but we shall keep q general for the moment. The angular momentum quantum number l takes the values $l=q,\ q+1,\ \ldots$ and the magnetic quantum number $m=-l,-l+1,\ldots,l$.

Although the final result eq.(2.10) is infrared finite, each one of the two terms has a part proportional to the volume. It is thus convenient to introduce an infrared regulator. We choose to do this by placing our quantum mechanical system in the potential of a spherically symmetric harmonic oscillator

$$\mathcal{V}(r)=\frac{1}{2}\,\omega r^2\,. \tag{2.14}$$

The thermodynamic limit is recovered as $\omega\to 0$.

The radial wave function $f_l(r)$ satisfies the equation

$$\left[-\left(\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}\right)+\frac{l(l+1)-q^2}{r^2}+\frac{1}{2}\,\omega r^2\right]f_l(r)=\epsilon f_l(r)\,. \tag{2.15}$$

The spectrum of eq.(2.15) can be found in a similar way to the well known case of the ordinary spherically symmetric oscillator problem (see e.g. [13]). Requiring that the radial function $f_l(r)$ vanishes at infinity, we have:

$$\epsilon^{(\pm)}_{n,l}=\omega(2n+2\rho^{(\pm)}_l+\tfrac{1}{2})\,, \tag{2.16}$$

with integers $n = 0, 1, 2, \ldots$ and parameters

$$\rho_l^{(\pm)} = \frac{1}{4} \pm \frac{1}{2}\sqrt{(l+\tfrac{1}{2})^2 - q^2}\,. \tag{2.17}$$

The parameters $\rho_l^{(\pm)}$ control the behaviour of the radial wave function f_l at the origin:

$$f_l(r) \;\propto\; r^{2\rho_l^{(\pm)}-1} \text{ as } r \to 0\,. \tag{2.18}$$

In order to ensure that the Hamiltonian is self-adjoint, we demand the boundary condition that $f_l(r)$ is finite at the origin. This requirement is satisfied only with the $\rho_l^{(+)}$ branch of the solutions (2.17). Therefore we take as the spectrum for our problem $\epsilon_{n,l} = \epsilon_{n,l}^{(+)}$ and consider the expression for the powers of the resolvent

$$\mathrm{tr}\left(\frac{1}{-|D|^2 + M^2}\right)^s = \sum_{n=0}^{\infty}\sum_{l=q}^{\infty} \frac{2l+1}{[\epsilon_{n,l} + M^2]^s}\,, \tag{2.19}$$

where an appropriate power s has been introduced so as to ensure the absolute convergence of the infinite sums at intermediate stages in the calculation. Here the factor $2l+1$ in the numerator is the degeneracy due to the magnetic quantum number m. We can rewrite eq.(2.19) as

$$\sum_{n=0}^{\infty}\sum_{l=q}^{\infty} \frac{2l+1}{[\epsilon_{n,l} + M^2]^s} = \frac{2}{(2\omega)^s}\sum_{n=0}^{\infty}\sum_{l=0}^{\infty} \frac{(l+\frac{1}{2})(1+x)}{\left[n + \frac{1}{2} + \frac{1}{2}(l+\frac{1}{2})\sqrt{1+2x} + \frac{M^2}{2\omega}\right]^s}\,, \tag{2.20}$$

with

$$x = \frac{q}{l+\frac{1}{2}}\,. \tag{2.21}$$

For our purposes it is convenient to expand this expression in powers of x. We only need to keep terms up to and including order x^3. The reason is that each power of x makes the summation over l and n more convergent. Thus an expansion in powers of x under the summation sign is related to the expansion of the result of the summation in powers of ω/M^2. In fact, as we shall see below, each additional power of x leads to at least one additional power of ω/M^2. The term of order x^0 results in the sum of order ω^{-3} (this term diverges in the limit $\omega \to 0$ and has to cancel against the second term in eq.(2.10)). Thus all terms starting with x^4 vanish in the thermodynamic limit $\omega \to 0$. The only relevant terms in eq.(2.20) are therefore

$$\begin{aligned}\frac{2}{(2\omega)^s}\sum_{n=0}^{\infty}\sum_{l=0}^{\infty} \quad & \frac{(l+\frac{1}{2})}{\left[n+\frac{3}{4}+\frac{l}{2}+\frac{M^2}{2\omega}\right]^s}\left[1 + q\left(-\frac{s}{2[n+\frac{3}{4}+\frac{l}{2}+\frac{M^2}{2\omega}]} + \frac{1}{l+\frac{1}{2}}\right)\right. \\ & + q^2\left(-\frac{s}{4[n+\frac{3}{4}+\frac{l}{2}+\frac{M^2}{2\omega}](l+\frac{1}{2})} + \frac{s(s+1)}{8[n+\frac{3}{4}+\frac{l}{2}+\frac{M^2}{2\omega}]^2}\right) \\ & \left. + q^3\left(-\frac{s(s+1)(s+2)}{48[n+\frac{3}{4}+\frac{l}{2}+\frac{M^2}{2\omega}]^3}\right)\right]\,.\end{aligned} \tag{2.22}$$

Note that, as expected, the first term in this expression exactly cancels the second term in eq.(2.10). To calculate the rest of the terms we use the following two basic integrals

$$\sum_{n=0}^{\infty}\sum_{l=0}^{\infty}\frac{(l+\frac{1}{2})}{\left[n+\frac{3}{4}+\frac{l}{2}+\frac{M^2}{2\omega}\right]^p}=\frac{1}{16}\frac{1}{\Gamma(p)}\int_0^\infty dt\, t^{p-1}e^{-\frac{M^2}{2\omega}t}\frac{1}{\sinh^3(\frac{t}{4})}\,, \tag{2.23}$$

$$\sum_{n=0}^{\infty}\sum_{l=0}^{\infty}\frac{1}{\left[n+\frac{3}{4}+\frac{l}{2}+\frac{M^2}{2\omega}\right]^p}=\frac{1}{8}\frac{1}{\Gamma(p)}\int_0^\infty dt\, t^{p-1}e^{-\frac{M^2}{2\omega}t}\frac{1}{\cosh(\frac{t}{4})\sinh^2(\frac{t}{4})}\,. \tag{2.24}$$

These expressions follows from the well-known infinite integral representation for the generalized Riemann zeta function (see e.g. [14]).

Expansion in powers of ω/M^2 is simply achieved by expanding the factors that multiply $\exp\{-\frac{M^2}{2\omega}t\}$ in the integrands in eqs.(2.24) in powers of t. Clearly increasing the power p by one or adding a factor of $l+\frac{1}{2}$ in the denominator in these expressions, leads to an extra power of ω/M^2. From eq.(2.22), each additional power of q (and hence x) in the expansion is indeed accompanied by such an increase and thus results in a higher power of ω/M^2 as claimed above. Some simple algebra shows that the terms $O(\omega^{-2})$ and $O(\omega^{-1})$ in eq.(2.22) vanish, and the final result to order $O(\omega^0)$ is

$$-\frac{1}{6}q(q^2+\frac{1}{2})M^{-2s}\,. \tag{2.25}$$

Now taking $s=1$, $q=1/2$ and referring back to eq.(2.10) we obtain

$$\alpha=\frac{1}{16}\,. \tag{2.26}$$

Putting these results together, we find that to one-loop order the vacuum expectation value of the vortex operator is

$$\langle V\rangle = e^{-\frac{\pi}{2g^2}\Lambda}\left(\frac{M^2}{\Lambda^2}\right)^{\frac{1}{16}}\,. \tag{2.27}$$

Although we have not calculated directly the correlation function of the vortex operator, eq.(2.27) in conjunction with the mechanics of the calculation allow us to understand its main features. The calculation of the correlation function would lead to the quantum mechanical problem of a particle in the background of the monopole-antimonopole pair separated by a distance $r=|x-y|$. Clearly, if the separation of the pair is larger than the inverse mass of the Higgs field, the result of the calculation will simply be the square of eq.(2.27). On the other hand, if the distance is much smaller that M^{-1}, the Higgs mass will not enter the final result. Instead the mass M will be substituted by r^{-1}. Thus the correlation function has the form

$$\langle V(x)V^*(y)\rangle = e^{-\frac{\pi}{g^2}[\Lambda-\frac{1}{|x-y|}]}\left(\frac{M^2}{\Lambda^2}\right)^{\frac{1}{8}}f(M^2r^2)\,, \tag{2.28}$$

where

$$\lim_{z\to\infty}f(z)=1\,, \tag{2.29}$$

$$\lim_{z\to 0}f(z)\;\propto\; z^{-1/8}\,. \tag{2.30}$$

We thus find that the presence of the dynamical Higgs field leads to an additional factor $|x-y|^{-1/4}$ in the vortex-antivortex correlator in the ultraviolet region.

3. The $SU(2)$ Georgi-Glashow model

Next we consider the $SU(2)$ gauge theory with an adjoint Higgs field

$$L = \frac{1}{4}(F^a_{\mu\nu})^2 + (D^{ab}_\mu \phi^b)^2 + M^2 \phi^a \phi^a \,. \tag{3.1}$$

The vortex operator in this theory is defined as

$$V(x) = \exp\left\{\frac{2\pi i}{g}\int_{C(x)} E^a_i(y)\hat{\phi}^a(y)\epsilon_{ij}dy_j\right\} \tag{3.2}$$

with the unit vector

$$\hat{\phi}^a = \frac{\phi^a}{|\phi|}\,. \tag{3.3}$$

As discussed in detail in [7], the theory is invariant under the Z_2 magnetic symmetry, $V(x) \to -V(x)$. The nonvanishing expectation value of $V(x)$ breaks this symmetry spontaneously and the magnetic symmetry breaking is tantamount to linear confinement.

Although the calculation of $\langle V\rangle$ has been discussed in some detail in the framework of the effective low energy theory [7], we are not aware of its direct calculation in the microscopic theory defined by the Lagrangian eq.(3.1). We therefore start our discussion by setting up this calculation. In the path integral formalism

$$\langle V(x)\rangle = Z^{-1}\int [dA^a_\mu][d\phi^a] \exp - \int d^3x \left(\frac{1}{4}(F^a_{\mu\nu} - \hat{\phi}^a s_{\mu\nu})^2 + (D\phi)^2 + M^2\phi^2\right)\,. \tag{3.4}$$

As in the case of QED, in the leading order the Higgs field is irrelevant and we must minimize the pure Yang-Mills action in the presence of the source $\hat{\phi}^a s_{\mu\nu}$. To do this, we first choose $\hat{\phi}^a = \delta^{a3}$. In the context of the present calculation this can be considered as a gauge fixing. Having found the classical solution for this source, b^a_μ, the solution for a general $\hat{\phi}$ is found by gauge transforming it:

$$A^a_{\mu\text{cl}} = U^\dagger b_\mu U + \frac{i}{g}U^\dagger \partial_\mu U\,, \tag{3.5}$$

where the matrix U is determined by the Higgs field via

$$\sigma^a\hat{\phi}^a = U^\dagger \sigma^3 U \tag{3.6}$$

with σ^a the Pauli matrices.

The problem of minimizing the action with the source is similar to that in the Abelian theory. There it is known that the solution is a pointlike Dirac monopole. In the present case this is not entirely obvious. After all, we know that the $SU(2)$ theory has finite action solutions for monopoles with magnetic charge double that of the elementary Dirac monopoles. If this also were true for the case at hand, such a finite action solution would

be preferred over the pointlike Dirac monopole whose action is linearly divergent in the UV. To put it another way, we know from the effective theory calculations in the Higgs regime [7], that the leading behaviour of the VEV in eq.(3.4) is $\exp\left(-2\pi\frac{\lambda}{g^2}\right)$. What the effective theory can not tell us is whether the scale λ is the UV cutoff of the effective theory (the mass of the W-boson M_W), or the genuine UV cutoff Λ. The only way to settle this question is to solve the classical equations of the underlying $SU(2)$ gauge theory.

We will show now that the classical solution which minimizes the action is the pointlike Dirac monopole, and that it has the action which diverges linearly in the infinite cutoff limit.

It is useful to perform the calculation using the spherical coordinates

$$x_1 = r\cos\phi\sin\theta, \quad x_2 = r\sin\phi\sin\theta, \quad x_3 = r\cos\theta\,. \tag{3.7}$$

It is known [15] that for monopoles with even magnetic charge classical solutions do not always have spherical symmetry, but are rather axially symmetric. We thus take for our solution the general (up to a gauge rotation around the third axis) axially symmetric ansatz

$$g\ b^a_\mu\ \sigma^a = A(\theta,r)\partial_\mu\phi\ \sigma^1_{(n\phi)} + B(\theta,r)\partial_\mu\theta\ \sigma^2_{(n\phi)} + C(\theta)\ \partial_\mu\phi\ \sigma^3\,, \tag{3.8}$$

where A, B and C are scalar functions, n is an integer and

$$\sigma^1_{(n\phi)} = \cos(n\phi)\ \sigma^1 + \sin(n\phi)\ \sigma^2\,, \quad \sigma^2_{(n\phi)} = \cos(n\phi)\ \sigma^2 - \sin(n\phi)\ \sigma^1\,. \tag{3.9}$$

The dual field strength $\tilde{F}^a_\mu = \frac{1}{2}\epsilon_{\mu\nu\lambda}F^a_{\nu\lambda}$ for this ansatz is

$$\begin{aligned} g\tilde{F}^a_\mu\ \sigma^a \;=\; & \Big(\pi\delta_{\mu 3}[\Theta(x_3)C(0) + \Theta(-x_3)C(\pi)]\delta(x_1)\delta(x_2) + [C' - AB]\epsilon_{\mu\nu\lambda}\partial_\lambda\phi\partial_\nu\theta\Big)\,\sigma^3 \\ & + \Big(\epsilon_{\mu\nu\lambda}\partial_\nu r\partial_\lambda\phi\frac{\partial A}{\partial r} + \epsilon_{\mu\nu\lambda}\partial_\nu\theta\partial_\lambda\phi\,[A' + (n+C)B]\Big)\,\sigma^1_{(n\phi)} \\ & + \epsilon_{\mu\nu\lambda}\partial_\nu r\partial_\lambda\theta\ \frac{\partial B}{\partial r}\,\sigma^2_{(n\phi)}\,. \end{aligned} \tag{3.10}$$

Here $\Theta(x)$ is a step function and the prime over a function denotes its derivative with respect to θ. Clearly, in order to cancel the string contribution in eq.(3.4) the function C must satisfy

$$C(\theta = 0) = 0\,, \qquad C(\theta = \pi) = 1\,. \tag{3.11}$$

For the sake of generality we will for now take the string contribution $s_{\mu\nu}$ in eq.(3.4) to have the strength ν, and thus take

$$C(\theta = \pi) = \nu\,. \tag{3.12}$$

Eventually we are interested in $\nu = 1$. In eq.(3.10) we have assumed that

$$A(r,\theta = 0) = 0\,, \qquad A(r,\theta = \pi) = 0\,, \tag{3.13}$$

as otherwise there would be an extra string like singularity along the x_3 axis in a $\sigma^1_{(n\phi)}$ component of the field strength.

Note, that in order to have the action of this configuration ultraviolet finite, one needs

$$\left[C'(\theta) - A(r,\theta)B(r,\theta)\right]|_{r=0} = 0\,. \tag{3.14}$$

However, this condition can be satisfied only for an even values of ν. Therefore, instead of imposing this condition we calculate the action for the ansatz eq.(3.8) and minimize it with respect to variational functions A, B and C as well as the integer parameter n. The leading piece in the action is the UV divergent integral

$$S \sim \Lambda \int_0^\pi \frac{d\theta}{\sin\theta}\{[C' - AB]^2 + [B(C+n) + A']^2\}\,, \tag{3.15}$$

where the functions A and B are evaluated at $r = 0$. It turns out that the minimization equations with respect to A and B can be solved exactly at given C (see Appendix). The solution is

$$B = \frac{AC' - A'(C+n)}{A^2 + (C+n)^2}\,, \tag{3.16}$$

$$A^2 + (C+n)^2 = (1 - a\cos\theta)^2 b^2\,. \tag{3.17}$$

Imposing the boundary conditions at $\theta = 0$ and $\theta = \pi$, we find that

$$n^2 = (1-a)^2 b^2\,, \tag{3.18}$$

$$(\nu + n)^2 = (1+a)^2 b^2\,. \tag{3.19}$$

There are two types of solutions to these:

$$a = \frac{\nu}{2n+\nu}, \qquad b = \pm\frac{2n+\nu}{2}, \tag{3.20}$$

and

$$a = \frac{2n+\nu}{\nu}, \qquad b = \pm\frac{\nu}{2}\,. \tag{3.21}$$

The divergent part of the action can now be calculated for both solutions. It turns out to be independent of $C(\theta)$

$$S \sim 2a^2b^2\Lambda\,. \tag{3.22}$$

For the two possible solutions

$$S_1 = \frac{1}{2}\,\nu^2\,\Lambda\,, \tag{3.23}$$

$$S_2 = \frac{1}{2}\,(2n+\nu)^2\,\Lambda\,. \tag{3.24}$$

Thus for even values of ν, which correspond to 't Hooft-Polyakov monopoles, the action is minimized by choosing the solution eq.(3.21) and $n = -\nu/2$, and it is UV finite. On the other hand in the case of interest to us, $\nu = 1$, the action is minimized with eq.(3.20) for any n as well as for eq.(3.21) with $n = 0$ and $n = -1$, but is UV divergent.

Note that for $\nu = 1$ a simple representative of the class of configurations with minimum action is given by

$$C(\theta) = \frac{1}{2}(1 - \cos\theta), \quad A = 0, \quad B = 0\,. \tag{3.25}$$

This is precisely the abelian Dirac monopole solution[1].

We conclude from this discussion that a classical configuration that minimizes the action in the unitary gauge is the abelian Dirac monopole

$$b^a_\mu = \frac{1}{g}\,\delta^{a3}(1-\cos\theta)\,\partial_\mu\phi\,. \tag{3.26}$$

The action for this configuration is the same as in QED, as all the nonabelian components of the gauge field vanish. The VEV of the vortex operator in the Georgi-Glashow model in the leading perturbative order is thus

$$\langle V\rangle_0 = e^{-\frac{\pi}{2g^2}\Lambda}\,. \tag{3.27}$$

Our next step is to consider the one loop corrections. To this end we write the vector potential as the sum of the classical solution and the fluctuation field

$$A^a_\mu = A^a_{\mathrm{cl}\mu} + a^a_\mu\,, \tag{3.28}$$

with $A^a_{\mathrm{cl}\mu}$ defined in eq.(3.5). In this expansion we consider the classical field to be of order $1/g$, while the fluctuation field to be of order one. The Higgs field is also considered to be of order one. For a one loop calculation we need to expand the action in eq.(3.4) to order one. Note that the relation between the classical solution and the Higgs field is such that

$$D^{ab}_\mu(A_{\mathrm{cl}})\hat\phi^b = 0\,. \tag{3.29}$$

Writing the Higgs field in terms of its modulus ρ and the unit vector $\hat\phi^a$ we find that

$$D^{ab}_\mu(A_{\mathrm{cl}})\phi^b = \hat\phi^a\partial_\mu\rho\,. \tag{3.30}$$

Thus to order one the action in eq.(3.4) is

$$\frac{\pi}{2g^2}\Lambda + \frac{1}{2}(D^{ab}_{[\mu}(A_{\mathrm{cl}})a^b_{\nu]})^2 + \epsilon^{abc}F^a_{\mu\nu}(A_{\mathrm{cl}})a^b_\mu a^c_\nu + (\partial_\mu\rho)^2 + M^2\rho^2 \tag{3.31}$$

Although the classical vector potential A_{cl} depends on the direction of the field ϕ, the integral over a^a_μ in eq.(3.4) does not depend on it. Thus the integration over a^a_μ and ϕ factorizes. The part of the action that contains ϕ does not know anything about the monopole field and thus is unaffected by the presence of the vortex operator in the path integral.

One should be a little careful with the analysis just presented, as it involves a perturbative calculation in the glue sector. This part of the calculation is in fact infrared divergent. The integration over the vector potential a^a_μ is very similar to the integration over the Higgs field in the QED case — both are charged fields coupled to a monopole. Thus the result of the integration over a^a_μ will be formally similar to eq.(2.27), except that

[1] Although our minimization procedure has only established that A and B vanish at the origin, it is a straightforward matter to show that in this case they will also vanish at all values of r. Any nontrivial r-dependence of either A or B immediately increases the energy due to the square of the last two terms in eq.(3.10).

the power in the last factor may be somewhat different. The important difference though is that the vector field is massless, and so instead of M as in eq.(2.27), here we will in fact have zero. This zero is the infrared divergence of the one loop correction to the monopole action.

This divergence however does not invalidate our conclusion. An expectation value of any local operator is not calculable perturbatively in a Yang-Mills theory, because it inevitably involves the knowledge of the infrared modes which are nonperturbative. However one can certainly calculate perturbatively a correlation function of any two such operators as long as the separation between them is smaller than the inverse coupling constant. If instead of an expectation value $\langle V\rangle$ we consider the correlation function $\langle V^*(x)V(y)\rangle$, the infrared cutoff in the calculation of the one loop correction will be provided by the separation $|x-y|$. On the other hand this will not change any of the important features of our calculation. The leading classical configuration will still be Abelian — this time an Abelian monopole-antimonopole pair. For this classical background eq.(3.29) still holds, and thus the Higgs field decouples from the background. The integration over the Higgs field therefore does not involve the background and the result knows nothing about $|x-y|$

$$\langle V(x)V^*(y)\rangle = e^{-\frac{\pi}{g^2}[\Lambda-\frac{1}{|x-y|}]}\left(\frac{1}{\Lambda^2(x-y)^2}\right)^{\gamma}, \tag{3.32}$$

where the constant γ is determined by the integration over the fluctuations of the gluon field. Thus the vortex-antivortex correlation function is not affected by the Higgs field to one loop order, as long as perturbation theory can be applied, i.e. $g^2 \ll M$, $g^2 \ll |x-y|^{-1}$.

We conclude this discussion with the observation that in the infinite Higgs mass limit, the Higgs field can be integrated exactly in the path integral eq.(3.4). In this limit the Higgs integral is dominated by the field configurations with vanishingly small action. The only appearance of the Higgs then is in the source term $\hat{\phi}^a s_{\mu\nu}$. The Higgs integral then degenerates into the form that allows the explicit calculation using the expression for the so-called Harish-Chandra-Itzykson-Zuber integral [16],[17] over the unitary group $U(N)$:

$$\int_U [dU] e^{\frac{1}{g}\mathrm{tr}(AUBU)} = (\prod_{n=1}^{N-1} n!)(\frac{1}{g})^{-\frac{N(N-1)}{2}} \frac{\det ||e^{\frac{1}{g}a_i b_j}||}{\Delta(a)\Delta(b)}, \tag{3.33}$$

where DU is the Haar measure, A and B are hermitian $N\times N$ matrices, $\Delta(a)$ and $\Delta(b)$ are Vandermonde determinants expressed in terms of the corresponding eigenvalues a_i and b_i

$$\Delta(a) = \prod_{1<i<j<N} (a_i - a_j). \tag{3.34}$$

Exploiting the representation (3.6) for the unit vector Higgs field one can integrate over $\hat{\phi}^a$ in eq.(3.4)

$$\langle V\rangle = Z^{-1}\int [dA_\mu] \exp - \int d^3x\, \frac{1}{4}\left\{F^2 + s^2\left(1 - \frac{g^2}{2\pi^2\Lambda}\ln\left[\frac{4\pi\sinh\mathcal{F}}{\mathcal{F}}\right]\right)\right\}, \tag{3.35}$$

where

$$\mathcal{F} = \frac{4\pi}{g\Lambda}\sqrt{\tilde{F}^a_\mu \tau_\mu(C)\tilde{F}^a_\nu \tau_\nu(C)}. \tag{3.36}$$

In eq.(3.35) the UV cutoff Λ is understood as the inverse of the discretization scale. This representation for the vacuum expectation value of vortex operators can be further simplified, by noting that finite contributions to the path integral come only from configurations for which $F \sim (\Lambda^2/g)$, since only those have a chance of cancelling the UV and IR divergence coming from the s^2 term. For these fields only the positive exponent in sinh has to be kept. Thus we arrive at

$$\langle V \rangle = Z^{-1} \int [dA_\mu] \exp - \int d^3x \, \frac{1}{4} \left\{ F^2 + s^2 \left(1 - \frac{g^2}{2\pi^2 \Lambda} \left[\mathcal{F} - \ln \frac{\mathcal{F}}{2\pi} \right] \right) \right\} . \tag{3.37}$$

This is the explicit gauge invariant expression for the vortex operator in pure gluodynamics in 2+1 dimensions.

4. Conclusions

In this paper we have investigated the influence of dynamical matter fields on the ultraviolet behaviour of the vacuum expectation value and correlator of vortex operators. We have done this for two models in 2+1 dimensions and have seen very different effects. For noncompact QED with a charged scalar field we have seen how the matter fields induces power like factors in the VEV and correlator. Thus the ultraviolet behaviour of the vortex operator in the theory with dynamical Higgs is different from that in the theory without the Higgs particle at distance scales $|r| < M^{-1}$. In contrast, for the $SU(2)$ gauge theory with the Higgs field in the adjoint representation, we have seen that the matter fields have no effect at one-loop order in the perturbative regime. In this theory, therefore, for $M \gg g^2$ the vortex operator is insensitive to the presence of the dynamical Higgs as soon as $|r| \ll g^{-2}$.

This is somewhat surprising, since generically one expects the presence of extra degrees of freedom to affect all observables. We may speculate that this is possibly related to another unexpected observation that holds in the same theory. Namely, it has been noted [18] that the spectrum of the $2+1$ dimensional $SU(2)$ gauge theory with adjoint Higgs is separated into two very distinct parts. One part contains glueballs, whose masses are almost completely independent of the Higgs mass as long as the Higgs is heavier than the gauge coupling. The other part of the spectrum contains "bound states" of the Higgs boson, and scales appropriately with the Higgs mass. The fact that the glueballs are not affected by the mass of the Higgs does not have a simple and natural explanation. Our finding is akin to this effect and suggests that the vortex operator correlation functions are heavily dominated by the glueball intermediate states.

Acknowledgments

A. Kh. thanks the School of Mathematics and Statistics at the University of Plymouth for their kind hospitality.

A. Appendix

In this appendix we perform the minimization of the UV divergent part of the monopole action:

$$\mathcal{S}^{UV} = \int \frac{d\theta}{\sin\theta} \left((C' - AB)^2 + (B(C+n) + A')^2\right) . \tag{A.1}$$

We now keep the function $C(\theta)$ fixed and minimize this functional with respect to A and B:

$$\frac{\delta \mathcal{S}^{UV}}{\delta A} = \frac{B(C' - AB)}{\sin\theta} + \frac{d}{d\theta}\left[\frac{A' + B(C+n)}{\sin\theta}\right] = 0\,, \tag{A.2}$$

$$\frac{\delta \mathcal{S}^{UV}}{\delta B} = A(C' - AB) - (C+n)(A' + B(C+n)) = 0\,. \tag{A.3}$$

Expressing B from eq.(A.3)

$$B = \frac{AC' - A(C+n)}{A^2 + (C+n)^2}\,, \tag{A.4}$$

and substituting this expression into the first equation (A.2) we find after some simple algebra:

$$\frac{(C+n)(AC' - A'(C+n))}{A^2 + (C+n)^2} + A' = A\frac{C'(C+n) + AA'}{A^2 + (C+n)^2}\,. \tag{A.5}$$

Now defining

$$Y = \frac{d}{d\theta} \ln\left[A^2 + (C+n)\right]\,, \tag{A.6}$$

we can write (A.5) in the form of a differential equation for the unknown function $Y(\theta)$:

$$\frac{dY}{d\theta} + \frac{1}{2}\, Y^2 - \cot\theta\; Y = 0\,. \tag{A.7}$$

In order to integrate this non-linear differential equation we introduce a new function Z defined by

$$Z = \cot\theta\; Y\,, \tag{A.8}$$

for which the equation $(A.7)$ reduces to the simple form

$$\frac{dZ}{d\theta} + (Z + \frac{1}{2}\, Z^2)\tan\theta = 0\,. \tag{A.9}$$

The general solution to this equation is

$$Z(\theta) = \frac{2a\cos\theta}{1 - \cos\theta}\,, \tag{A.10}$$

where a is an integration constant. From this solution one finds $\ln\left[A^2 + (C+n)\right]$ by integrating Y in $(A.6)$:

$$A^2 + (C+n)^2 = (1 - a\cos\theta)^2 b^2\,, \tag{A.11}$$

where b is a new constant of integration. Note that the constants a and b are subject of the equations that follows from the boundary conditions eqs.(3.11) and (3.13).

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